Activity 9 Winning and Losing the Lottery

Objectives

- Apply the probability formula
- Define *lottery*
- Use basic counting processes to find permutations or combinations in a given situation
- Determine the probability of possible combinations
- Compare the probability of winning a lottery with the probabilities of other events

Materials	paper, pencils, calculators, 250 push pins or thumbtacks, wall map of United States
Time	45 minutes
Math Idea	It is difficult to find an event that is less likely to occur than winning the lottery.

Prior Understanding

Students should know how to convert among fractions, decimals, and percents, as well as find probabilities using the probability formula. They should also know how to use the fundamental principle of counting, understand factorial notation, and differentiate between permutations and combinations.

Introduction: Gambling Connection

You can use or adapt the following as an introduction to the problem. Make a list of students' responses on the board. After students complete the activity, look back at the list and discuss any changes in students' responses.

Most state lotteries use some variation of the following procedure: players pick six numbers out of a pool of numbers such as 1 through 48 inclusively. On the scheduled day, a lottery official randomly draws six numbers from the same pool. If any player matches the six numbers drawn randomly, that player wins the jackpot. If more than one player matches the six random numbers, then the winners share the jackpot. If no one matches all six random numbers, the jackpot "rolls over" and is increased for the next drawing. What are the chances of winning and losing a lottery?

Exercise 1

Distribute copies of BLM 9 to students. Have them guess at the order of likelihood in which the given events might occur. Assign individuals or groups to research the likelihood of each event except winning the lottery.

Discussion

The events, in order from most likely to least likely, are as follows (all probabilities, except for the **lottery**, are average annual rates based on actual mortality rates):

Being killed in a car accident	one in 5,300
Being a drowning victim	one in 20,000
Choking to death	one in 68,000
Being killed in a bicycle accident	one in 75,000
Being killed by a terrorist in a foreign country	one in 1.6 million
Being killed by lightning	one in 2 million
Dying from a bee sting	one in 6 million
Winning the lottery	one in 12.3 million

Exercise 2

Have students find the total number of different ways to pick three out of four numbers: 1, 2, 3, 4. Then have them find the probability that any one combination of three randomly drawn numbers will be chosen.

Discussion

Students can use a tree diagram to find the 24 possible ways of choosing three out of four numbers, of which only four are distinct from the others (1, 2, 3; 1, 2, 4; 1, 3, 4; 2, 3, 4). This is the same as calculating the **combination** C(4, 3) = $\frac{4!}{3!1!} = \frac{4\times3\times2\times1}{(3\times2\times1)(1)} = 4$. Each **combination** of three numbers has a 1/4 or 25% possibility of being drawn.

Exercise 3

Have students start with the numbers from 1 to 7 and find the probability of winning by choosing six out of seven numbers in a random draw.

Discussion

There are seven ways to choose six out of seven numbers (1, 2, 3, 4, 5, 6; 1, 2, 3, 4, 5, 7; 1, 2, 3, 4, 6, 7; 1, 2, 3, 5, 6, 7; 1, 3, 4, 5, 6, 7; 1, 2, 4, 5, 6, 7; 2, 3, 4, 5, 6, 7); and a 1/7 or 14.3% probability of any one of the **combinations** being drawn.

Exercise 4

Have students calculate the probability of choosing a winning combination with:

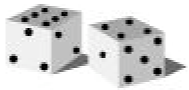
(a) 6 out of 10 numbers (1 to 10)

(b) 6 out of 20 numbers (1 to 20)

(c) 6 out of 48 numbers (1 to 48)

Using the result obtained for 6 out of 48 numbers, have students assume that that many tickets are sold. Discuss whether or not a winner would be guaranteed.

If all possible combinations of 6 out of 48 numbers are selected and no one buys more than one ticket, have students determine the number of people that would *lose* the lottery drawing.



Discussion

For 6 out of 10 numbers, students should calculate C(10, 6) = $\frac{10!}{6!4!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$ **combinations** with a 1 out of 210 (or 0.47%) chance of winning. For 6 out of 20 numbers, students should calculate C(20, 6) = $\frac{20!}{6!14!} = 38{,}760$ **combinations** with a 1 out of 38,760 (or 0.0025%) chance of winning.

For 6 out of 48 numbers, C(48, 6) = $\frac{48!}{6!42!} = 12{,}271{,}512$ **combinations** with a 1 out of 12,271,512 (or 0.0000081%) chance of winning.

To put this figure into perspective, assume there are about 4 million people in Louisiana. Suppose Louisiana had a **lottery** game that drew 6 numbers out of a pool of 48. Even if every person in the state bought 3 tickets each, there would be no guarantee that any of the 4 million people would win the **lottery**, because 12 million tickets would not cover all of the 12.3 million possible **combinations** of six numbers. Furthermore, even if 12.3 million tickets were sold, there still would be no guarantee that anyone would win, since the same losing **combination** of numbers could have been selected by more than one person. In this case, all of the 12.3 million possible number **combinations** would not have been covered, and the winning number could be one of the **combinations** not purchased.

Students should also note that none of the other events listed on BLM 9 comes close to the tiny probability of winning a state **lottery**. Nevertheless, many people probably consider winning the **lottery** much more likely than, for example, dying in a car crash. And the lottery never publicizes the fact that a person is almost six times more likely to be killed by lightning than to win the **lottery**.

Students should realize that there would be 12,271,511 losers. To give students an idea of how big a number this is, have them assume a 1-to-1 correspondence between the number of losers and number of middle school students in the United States. Ask them to find the number of

(a) classrooms full of losers (assume 30 students per class) *(12,217,511/30 = 409,050 classrooms)*

(b) the number of schools full of losers (assume 40 classes per school) *(409,050/40 = 10,226 schools)*

(c) the number of schools in every state completely full of losers *(10,266/50 = 205 schools per state).*

Using a map of the United States, have students try to put 205 push pins in just one state—the state will be full of losers. Remind the students that each push pin represents a school of 1,200 students, just like their school, and that every state in the country would have 205 push pins in it just from a single **lottery** drawing!

Activity 9 Winning and Losing the Lottery Teacher Support

Vocabulary

combination a selection of things in which order does not matter

fundamental principle of counting the number of ways of making several successive decisions is the product of the number of choices that can be made in each decision

lottery gambling game in which players buy tickets bearing combinations of numbers that must match a combination of numbers selected at random in order to win

permutation an arrangement of things in a definite order

probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

Ongoing Assessment

Have students calculate the chance of winning a pick 6 out of 49 lottery. *(1 out of 13,983,816 or about 1 in 14 million)*

Added Practice 9 Winning and Losing the Lottery

Name ______________________________**Date**______________________

The media always focus on the single lottery winner but neglect to mention the vast number of losers. So, imagine you had a 3" × 4" photograph of every *loser* in a single lottery drawing.

1. Suppose you build a scaffolding box around the Statue of Liberty and want to cover the surface of the box with pictures of losers. The dimensions are given below. Calculate the surface area of the box (assume the box has no top or bottom).

Statue:	height—151 feet
Base:	height—154 feet
	width—154 feet
	depth—154 feet

2. Determine how many photos are needed to cover one square foot of surface area and the number of photos needed to cover the entire scaffolding.

3. Determine how many Statues of Liberty you could cover with the 12,271,511 photos of losers.

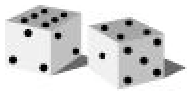
     

Answer Key Added Practice 9 Winning and Losing the Lottery

1. Surface Area = (height of Statue + height of base) × (width of base) × 4 = (151' + 154') × 154' × 4 = 187,880 sq. ft.
2. The area of each photo is (1/4’) × (1/3’) = 1/12 sq. ft. So 12 photos cover 1 square foot of surface area.

   187,880 sq. ft x 12 photos per sq. ft = 2,254,560 photos needed to cover the entire scaffolding
3. $\frac{12{,}271{,}511 \text{ photos of losers}}{2{,}254{,}560 \text{ photos per Statue}} = 5.44$ Statues of Liberty

   Can you imagine seeing five of those statues completely covered with pictures of lottery losers from just one drawing-- with enough pictures of losers left over to cover the sixth statue up to its waist?



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Name ______________________________Date______________________

Based on your knowledge or opinions, put the following events in order from most likely to least likely.