Activity 1 Thinking About Averages

Objectives

- Apply the concepts of mean, median, and mode
- Understand the concept of sampling
- Collect, organize, and display data in a variety of ways
- Use observations about differences between samples to make conjectures about the populations involved

Materials	paper, pencils, calculators
Time	30–45 minutes
Math Idea	The word *average* often connotes “typical,” “common,” “middle,” or “midpoint.” However, statistical averages have a more specific meaning.

Prior Understanding

Students should know how to find the mean, median, and mode for a set of values. They should also know how to calculate with decimals and convert from one measurement unit to another.

Introduction

The following scenario may be used as an introduction to the problem of thinking about averages.

A wealthy couple is planning a dinner party for 5 guests. Their ages are 89, 92, 17, 2, and 2. The couple gives a list of the guests and their ages to the butler, the disc jockey (DJ) in charge of entertainment, and the cook so that proper arrangements can be made. Based on "averages," what would you predict would be served at dinner? What kind of music should be played? Would you serve after-dinner drinks? Why or why not?

Discussion

After discussion of the scenario, tell students that at the party everyone was served strained peas at dinner accompanied by the latest CD from The Backstreet Boys, followed by cognac. Discuss how the three members of the staff could each have arrived at a different idea based on "averages."

Students might realize that the butler looked at the guest list and calculated the mean age: (89 + 92 + 17 + 2 + 2)/5 = 202/5 = 40.4, so served cognac after dinner. The DJ used the **median** age, 17, as the average, so picked appropriate music. The cook used the **mode** or most common age, 2, as the average and served strained peas. Although this scenario would probably not occur in real life, students should be aware of the different ways to discuss averages.

Exercise 1

Help students develop a simple data chart to collect information that could be used to find the “average” height for your class. Divide students into groups. Let each group decide how they will choose a sample and collect and record the data. Then have each group calculate the mean, median, and mode for its sample.

Discussion

Briefly, discuss ideas for a data collection chart. Two columns, one with a name, the other with a height, are sufficient. Students should be aware of the advantage of organizing their **data** in a manner that allows for easy calculation.

Each group of students will most likely choose a different **sample** (will ask a different combination of students), so expect that students will come up with a diverse range of **means**, **medians**, and **modes**. Allow 10 minutes for students to collect their data; some students will collect more **data** than others.

Discuss the problem of making calculations with heights that are recorded in mixed units, such as 5'2" and 4'8". If the heights were first converted into one unit, such as inches, the task is easier. After calculating the **mean**, **median**, and **mode**, students can convert each average back into feet and inches.

Help students realize that a **sample** is a small portion taken from the entire population, which would be every individual in the group of all possible observations. Often it is unrealistic to examine every member of a population, especially if that population is very large. In addition to being time-consuming, evaluating an entire population is often too expensive. For practical purposes, studies are usually conducted by selecting a **sample** from a larger population. To ensure that this smaller sample is representative of the larger population, the **sample** should be selected according to a random process. In random processes, each member of the population has the same chance of being selected.

Discuss whether students selected their **sample**s randomly. For example, a random sample might be every fifth name off the class roster listed alphabetically. Most students probably selected their friends, those in their group, or those who were sitting nearest to them. This might be a *biased sample*; if an 8th grader who is 5'6" sampled his or her friends, he or she may be more likely to have friends who also are tall.

Exercise 2

Compile data for the entire class on the board and have students find the mean, median, and modal heights for the class. Discuss differences between the total class results and the results from their samples. Let students speculate as to how the sample they used might have influenced their results.

Discussion

Students should realize that the only way to calculate the precise **mean**, **median**, and **mode** for the class is to include every student in the **sample**. However, if a random sample was chosen, the three measures they obtained might be good *estimates* for the class.

Activity 1 Thinking About Averages Teacher Support

Vocabulary

data information, often in the form of facts or figures obtained from experiments or surveys, used as a basis for making calculations or drawing conclusions

distribution the spread of statistics within known or possible limits

mean the arithmetic average of a set of numbers

median the middle value in a set of values that have been arranged in ascending or descending order; the midpoint, or the value in a distribution above and below which 50% of the values lie

mode the number that occurs most frequently in a set of numbers

sampling the process of selecting a group of people to be used as representative of an entire population

Ongoing Assessment

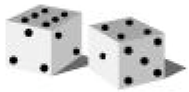
For any one mean, there are many possible **distributions** that could produce that mean. Have students estimate the **mean** for each of these three **distributions**:

(a) all the values are between 95 and 105

(b) half the values are around 50 and the other half are around 150

(c) one-quarter of the values are 0, half are near 50, and one-quarter are around 300

(All three distributions have the same mean—100; however, the other characteristics of these distributions are very different.)



Added Practice 1 Thinking About Averages

Name ______________________________ Date______________________

1. Suppose you want to survey residents of Baton Rouge to find out their favorite radio station. Would it make sense to sample every person who lives in Baton Rouge for your survey? Why or why not?

2. If you ask a sample of people living in Baton Rouge what their favorite radio station is, what could you do to select a sample whose answers would reflect that of the larger population?

3. Two American women, Florence Griffith Joyner and "FleetFoot," are competing in the 100-meter dash. Florence Griffith Joyner's past times were 11.08, 10.81, 10.75, 10.62 and 10.49 seconds, for an average of 10.75 seconds. FleetFoot's past times were 11.51, 11.25, and 11.89 seconds, for an average of 11.55 seconds. To calculate the qualifying time for the team, the officials find the mean time of these two runners. What is the team's mean time? What is the team's median time? What is the team's mode time?

Answer Key

Added Practice 1 Thinking About Averages

1. Students might say that it would be too time-consuming and/or too expensive to survey the entire population of Baton Rouge. By the time they finished asking everyone, the radio stations could have changed formats or no longer exist.

2. Students should address ideas about who to include or not include in the sample and how to pick a random sample rather than a convenient one (such as friends and relatives). Students should also examine whether the members of the selected group have any common characteristics, such as age, education level, income.

3. The average time for the team is 11.05 seconds. Students should add the values representing both runners' previous times and divide the sum by 8:

11.08 + 10.81 + 10.75 + 10.62 + 10.49 + 11.51 + 11.25 + 11.89 = 88.4; 88.4/8 = 11.05

A common mistake with this type of question is to average the averages, or

10.75 + 11.55 = 22.3; 22.3/2 = 11.15 seconds.

If both women had run the same number of races (four), then you could find the average time by averaging the two averages. However, Joyner ran five races and "FleetFoot" ran three races. To find Joyner's average (10.75), the five times were "condensed" into one; to find Fleetfoot's average (11.55) three times were "condensed" into one. Each of Joyner's five times is represented less in her average than each of Fleetfoot's three times is represented in her average. In other words, each of Joyner's times represents one fifth of her average, but each of Fleetfoot's times represents one third (a greater portion) of her average. Thus, if you average the averages, Fleetfoot's times are "weighted" more, or represent more, than Joyner's do in the final average.

To find the team's median time, first arrange the times in order: 10.49, 10.62, 10.75, 10.81, 11.08, 11.25, 11.51, 11.89. The middle value lies between 10.81 and 11.08: (10.81 + 11.08)/2 = 21.89/2= 10.95. There is no mode.