PROBABILITY AND GAMBLING

"Show me a gambler and I'll show you a loser."

Mario Puzo

"A stereotyped but unconscious despair is concealed even under what are called the games and amusements of mankind."

Henry David Thoreau

♣ ♦ ♥ ♠

Objectives

■ 2-1 The objectives for this section exposes students to the mathematics behind lotteries and the manner in which lottery numbers are chosen. After being presented the material from this section, students will better understand probability in the context of the lottery.

Objectives 2-1

- Exposes students to the mathematics behind lotteries
- Students will better understand probability in the context of the lottery

A Note on Probabilities Included in this Section

This section uses comparisons between the probability of winning the lottery and probabilities of other events to make a general point: it is difficult to find an event that is less likely than winning the lottery. This statement remains true even if you include events that seem incredibly rare. These comparisons are made to reinforce a conceptual understanding of probability and to improve students' number sense by providing frames of reference for the figures they will derive.

It is difficult to find an event that is less likely than winning the lottery.

However, as was demonstrated in Section 1, any probability derived from past events in the "real world" (as opposed to probability based on true random selection) is not precise. These figures are simply rough estimates that allow for general comparisons. These "real world" probabilities could be calculated in a number of ways, depending on what population you decide is a representative group and what individual factors you take into account. For example, this chapter gives the probability of dying in a bicycle accident as one in 75,000. This is an annual national rate. In other words, if you divide the number of bicycle accident deaths that occurred in this country during the year by the population of the country, you will get 1/75,000. Thus, the “average” risk of dying in a bicycle accident is one in 75,000 for each person in the country. A more complicated but more precise way to calculate the risk would be to divide the number of deaths by the number of people who rode a bicycle during the year. An even more precise way would be to derive a formula based on the number of hours spent riding a bicycle, the locations where the bicycle was ridden, safety equipment worn, and other factors.

Clearly, there are many factors that would affect an individual’s chances of being killed in a bicycle accident. It would be nearly impossible to create a formula that took into account every single factor and yielded an exact probability. For the purposes of this curriculum, we have used “average” rates to facilitate calculation and instruction. These rates will be accurate when examining the entire relevant population, but may be higher or lower than any individual’s actual chances (e.g., if you never ride a bicycle, your chances of dying in a bicycle accident are probably nearly zero). The main purpose of this section is to show how these probability estimates compare with the probability of winning the lottery.

Probability of Winning the Lottery

Most state lotteries use some variation of the following procedure: players pick six numbers out of a pool of numbers (e.g., 1 through 48). On the scheduled day, a lottery official randomly chooses six numbers from the same pool. If any player matches the six numbers drawn randomly, that player wins the jackpot. If no one matches all six numbers, the jackpot “rolls over” and is increased for the next drawing.

The chances of winning the lottery can be calculated using the principles of probability that were presented earlier in this module. The principles used for this calculation are the same regardless of the size of the pool from which numbers are drawn. Thus, although lotteries often use 48- or 49-number pools and draw a 6-number combination, these principles can be illustrated with a small pool of numbers. To simplify these concepts, we will describe this procedure with a 4-number pool. Suppose the lottery is drawing a three-number combination from a pool that includes the numbers 1, 2, 3, and 4.

2-2

Example: 4-number Pool

What is the probability of matching all three numbers?

Examine the probability of matching each of the three numbers separately

- 1st number: 1/4 (4 possible outcomes)
- 2nd number: 1/3 (3 remaining outcomes)
- 3rd number: 1/2 (2 remaining outcomes)

Multiply probabilities to get the probability of these three separate events 1/4 x 1/3 x 1/2 = 1/24

Chance of picking the same number as the lottery 1/24

■ 2-2 To calculate the probability of matching this three-number combination, we can separately examine probability of matching each of the three numbers. The probability of matching the first number is 1/4, since there are four possible outcomes for the first number (1, 2, 3, or 4). After the first number is selected by the lottery, there are three remaining numbers in the pool. Thus, since there are three possible outcomes for the second number, the probability of matching the second number is 1/3. After the second number has been drawn, two of the four numbers have been removed from the pool, so there are two possible outcomes for the third number.

Thus, the probability of matching the third number is 1/2. Since matching the first number, matching the second number, and matching the third number are all separate events, you have to multiply them to determine the probability of all of them occurring. Thus, the probability of matching all three numbers is 1/4 x 1/3 x 1/2, or 1/24. In other words, there are 24 possible outcomes for this drawing, so your chances of picking the same one as the lottery is 1/24. These 24 combinations are depicted in Figure 1 below.

The following explanation describes how to determine the number of possible combinations mathematically. For the first number drawn, there are four possibilities; for each of these four first-number options, there are three second-number options, resulting in 12 first- and second-number combinations; and for each of these 12 first- and second-number options, there are two third-number options. In other words, there are 4 x 3 x 2 = 24 different combinations.

Figure 1: Possible Three-Number Combinations Using 1, 2, 3, and 4

Possibilities of the 1st number		**1**			**2**			**3**			**4**	
Possibilities for combinations of 1st and 2nd number	1,2	1,3	1,4	2,1	2,3	2,4	3,1	3,2	3,4	4,1	4,2	4,3
Possibilities for combinations of 1st, 2nd, and 3rd number	1,2,3 1,2,4	1,3,2 1,3,4	1,4,2 1,4,3	2,1,3 2,1,4	2,3,1 2,3,4	2,4,1 2,4,3	3,1,2 3,1,4	3,2,1 3,2,4	3,4,1 3,4,2	4,1,2 4,1,3	4,2,1 4,2,3	4,3,1 4,3,2

However, the way lotteries are run, you do not have to match the three numbers in the exact order they are drawn: if the lottery draws 421, that combination is the same as 241, 412, and all of the other possible combinations of 1, 2, and 4. As a result, for the purposes of a lottery, there are not 24 different combinations in this example, since many of these combinations would be considered the same in a lottery drawing (e.g., 124 and 214). In other words, for each three-number combination, there will be a certain number of equivalent forms created by changing the order of the numbers. Thus, to find the number of different combinations *in the* *case of a lottery*, we have to determine how many different ways each three-number combination can be arranged. This procedure is similar to the process described above. The following steps show the number of different ways any three-number combination can be arranged: for the first number, any of the three numbers can be selected, so there are three different options for the first number. For each of these three first-number options, there are two different second-number options, resulting in six first- and second-number combinations. For each of these six first- and second-number options, there is one third-number option. Thus, there are 3 x 2 x 1 = 6 different ways to arrange a three-number combination. These steps are graphically represented in Figure 2 below.

Figure 2:
Possible Three-Number Combinations Using 1, 2, and 4

Possibilities of the 1st number	**1**	**2**	**4**
Possibilities for combinations of 1st and 2nd number	1,2 1,4	2,1 2,4	4,1 4,2
Possibilities for combinations of 1st, 2nd, and 3rd number	1,2,4 1,4,2	2,1,4 2,4,1	4,1,2 4,2,1

This process reveals that for each collection of three numbers, there are actually six combinations that are considered identical by the lottery. This means that of the 24 original combinations from Figure 1, there are 24 ÷ 6 = 4 different combinations for the purposes of a lottery. Thus, your chances of matching the number drawn in this hypothetical lottery is 1/4.

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This procedure can be applied to a real lottery as well. While this discussion at first may seem too complicated for students, it uses the same principles and steps as the discussion above. With a pool of 48 numbers, we can calculate the probability of six randomly selected numbers matching your six numbers using the formula for combined probability [probability of A and B = (probability of A) x (probability of B)]. The probability of matching the first number is 1/48. Once you have matched the first number, the probability that you will match the next number is 1/47 (you have already removed one number from the pool, so there are 47 left to choose from). Similarly, the probability of matching the third, fourth, fifth, and sixth numbers are 1/46, 1/45, 1/44, and 1/43 respectively. Since these are all different events, you must multiply them to determine the probability of all of them happening. The result of this operation is 1 in 8,835,488,640. However, this operation does not take into account the possibility that your first number could have been, for example, the fourth number selected randomly by the lottery official. In other words, the collection 1, 3, 5, 7, 9, 11 is the same as 1, 5, 7, 3, 11, 9: we are not concerned with the order the numbers are in. You do not have to match your first number with the lottery's first number, you only have to have the same six numbers in the end. With any six numbers, there are 720 ways to arrange them (6 x 5 x 4 x 3 x 2 x 1). Thus, we divide 8,835,488,640 by 720 to arrive at 12,271,512. In other words, you have one chance in about 12.3 million of winning this lottery. Another way to think about this probability is that you have a 12,271,511 in 12,271,512 chance of losing. In other words, you are 99.99999185% likely to lose.

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To put this figure into perspective, there are around 6 million people in Massachusetts. Suppose Massachusetts had a lottery game that drew 6 numbers out of a pool of 48. Even if every person in the state bought 2 tickets each, there would be no guarantee that any of the 6 million people would win the lottery, because 12 million tickets would not cover all of the 12.3 million possible outcomes. Even if 12.3 million tickets were sold, there still would be no guarantee that anyone would win, since the same losing combination of numbers could have been selected by more than one person. In this case, all of the 12.3 million possible number combinations would not have been covered, and the winning number could be one of the combinations not purchased.

This concept is illustrated by Mass Millions, the Massachusetts lottery game most similar to this example in which 6 numbers are chosen from a pool of 49 numbers. The probability of winning the jackpot in this game is one in nearly 14 million. Out of the 494 Mass Millions drawings held between the inception of the game in 1987 and the beginning of 1994, there were 77 jackpot winners. In other words, we can estimate that there was no jackpot winner in over 84% of the drawings (Massachusetts State Lottery Commission, 1994). In addition, if two or more people in a drawing select the winning numbers, they all split the jackpot. Thus, the 77 jackpot wins may have occurred in fewer than 77 drawings. To illustrate with a hypothetical example, if two people shared half of the jackpot wins, then 90% of the drawings resulted in no jackpot winner.*

* In this hypothetical example, there were 51 winning drawings, 26 of which were shared by 2 people and 25 of which were won by a single person. Thus there were 77 winners, and 443 out of 494 drawings (89.68%) had no winner.

Increasing or decreasing the size of the number pool has an unexpected effect on your chances of winning. When the size of the pool drops from 48 to 42, the chances of winning change from one in 12.3 million to one in 5.2 million. When the size of the pool increases from 48 to 54, the chances of winning decrease to one in 25.8 million. The probability of winning the lottery is not directly proportional to the quantity of numbers in the pool (Crites, 1994).

The Lottery: Comparisons and Misperceptions

To help students improve their understanding of probability, the probability of winning the lottery can be compared with the probability of other events. For example, all of the following events represent a one-in-a-million risk of death: driving a car for 60 miles without a seatbelt; driving a motorcycle for 5 miles (with a helmet); flying 10 hours in a commercial plane **; smoking two cigarettes (McGervey, 1986).

> **This one-in-a-million risk of death results from increased exposure to cosmic radiation, not from the chance of a crash. There is a one-in-a-million risk of some kind of accident for each airplane takeoff and landing (McGervey, 1986).

In other words, to take one of these examples, if you buy a lottery ticket and then drive from Boston to Worcester without wearing a seatbelt, your chances of dying in a car accident on your way to Worcester are over 12 times greater than your chances of winning the lottery.

In addition, there are many misconceptions about the lottery that result from poor understanding of probability. For example, most people think that a collection of numbers such as 3, 16, 24, 30, 37, 42 would have a better chance of being selected than collections such as 1, 3, 4, 6, 8, 9 or 34, 35, 36, 37, 38, 39. Most people probably believe that numbers spread out throughout the range would be more likely to be picked than a collection of numbers under 10 or a collection of consecutive numbers. While it is true that there are more collections of numbers spread out than there are collections of numbers under ten or collections of consecutive numbers, that is irrelevant as far as random selection is concerned. You don't get any prize for matching the *pattern* of the winning numbers. You don't get to bet whether the numbers will be consecutive or nonconsecutive. There are 12.3 million possible combinations of numbers; one of them is going to be selected randomly. In other words, each of the 12.3 million combinations has the same chance of being selected as any of the other combinations. The fact that there are more spread-out combinations than consecutive number combinations doesn't make your spread-out combination more likely to be drawn than any other combination (Crites, 1994).

Similarly, some people believe that it is best to pick the same six numbers every time. However, if their numbers were chosen while they were away on vacation, most people would pick a different combination of numbers the next time. People might make this decision thinking that the probability of the same six numbers being picked twice in a row were astronomically small. In reality, however, you are not betting that two lottery drawings in a row will both be your numbers. You are only betting that the *next* drawing will be your numbers. Like the coin tosses discussed earlier, lottery drawings are independent events: the probability of any one outcome is always the same, regardless of what has happened before. If you wanted to determine the probability of your numbers winning in the *next* two lottery drawings, you actually would multiply 1/12,271,512 times 1/12,271,512. However, if you wanted to determine the probability of your numbers winning given that they won the last time, the equation would look like this: 1 x 1/12,271,512 = 1/12,271,512. You are determining the probability that both event 1 and event 2 will happen, but since event 1 has already happened, it has a probability of 1.

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EXERCISES

♣ Red & Black Marbles

Example

Suppose you have a bag that contains 5 red marbles and 5 black marbles, and one of the red marbles is marked with an “X”. Suppose you are going to reach into the bag and pull out a marble at random.

- What is the probability of choosing a black marble? (50%).
- What is the probability of choosing the marble with the “X” on it? (10%).

Next, suppose you took out all of the red marbles except the one with the “X” and put in 4 black marbles, for a total of 9 black marbles.

- Now what is the probability of choosing the marble with the “X” on it? (still 10%).
- What is the probability of choosing a black marble? (90%).

Finally, suppose you took out all but one of the black marbles and put in 8 red marbles.

- Now what is the probability of choosing the one with the “X” on it? (still 10%).
- What is the probability of choosing a black marble? (10%).

This exercise illustrates a principle of probability that is relevant to playing the lottery. Many people believe that they can increase their chances of winning the lottery by selecting numbers that are spread out over the range, as opposed to numbers that are consecutive or numbers that are below a certain value. “Since there are more spread-out combinations,” a lottery player might think, “a spread-out combination is more likely to be chosen, so I will choose a spread-out combination and increase my chances of winning.” However, as we can see by examining our marble example, this reasoning is not correct. Once we select a *specific* item to place our bet on (in this case, a specific red marble), the compositions of the various *groups* within the pool of items become irrelevant

As we saw, the probability of choosing the red “X” marble was the same in each case, regardless of the compositions of “red” and “black” within the bag. In other words, the probability of picking a *specific* red marble (as opposed to picking *any* red marble) is the same whether 90% of the marbles in the bag are red or only 10% of the marbles are red. Similarly, once you have selected a specific number combination for the lottery, it doesn't matter whether there are millions of other combinations like it ("spread-out" numbers) or fewer than 50 other combinations like it (consecutive numbers). With the marble example, you were betting on *your specific* red marble, not just any red marble. When you choose a number combination for the lottery, you are betting on your specific combination, not all the combinations that are like yours in some way. Once you look at all of the items in the pool as individual choices (as opposed to members of a group), it becomes clear that each individual item has the same chance of being selected as any other individual item. In other words, people tend to group the number combinations together according to some characteristic that they think the number combinations have in common. However, when it comes to random selection, none of the number combinations has anything in common with any other number combination: each one is an individual combination out of 12.3 million combinations.

When you choose a number combination for the lottery, you are betting on your specific combination, not all the combinations that are like yours in some way.

♣ Losers on Liberty

Required Materials

Required materials: map of the U.S., 250 thumbtacks

Classroom Discussion

We have seen from our calculations that the probability of winning a lottery in which you pick 6 of 48 possible numbers is 1 chance in 12,271,512. We know from that exercise that even if 12,271,512 tickets are sold, a winner is not guaranteed because the same combination of numbers could be picked by more than one person. For this exercise we want to address the following question: Assuming that all possible combinations of numbers were selected and no one had more than one ticket, what would the likelihood be of someone winning?

The answer to this question is 100%: one person would be assured of winning. *One* winner.

This one person is always the focus of our attention. But what if we look at how many people did not win? **How many people lost in this one lottery drawing?**

The answer to this question is easy: 12,271,512 - 1 = 12,271,511. "Yeah, so?" you may be thinking — but just how many losers is 12,271,511?

An Illustration

Let's go from what we do know to what we don't know. Imagine that every loser got matched up to one middle school student: How many students are sitting in your classroom? How many students in your school? How many middle schools in your town? How many towns like yours would have to have every student in every classroom matched to a loser so that you could see all the losers from just one draw? Note: Numbers have been slightly rounded or averaged to ease calculations.

$$\frac{\text{12,271,511 losers}}{\text{30 students per class}} = \text{409,050 classrooms full of losers!}$$

$$\frac{\text{409,050 losers}}{\text{40 classes in each school}} = \text{10,226 schools full of losers!}$$

$$\frac{\text{10,226 schools}}{\text{50 states}} = \text{205 schools in every state, completely FULL of losers!}$$

Using a map of the U.S., try to put 205 thumbtacks in just one state — the state will be full of losers. Remind the students that each thumbtack represents a school of 1,200 students, just like their school, and that every state in the country would have 205 thumbtacks in it. Just from a single lottery drawing! We want you to feel, see, and know how big the numbers of losers is for each and every lottery drawing.

The following exercise is another way to bring the numbers to life:

Example

Ask if the students have seen pictures of lottery winners in the newspaper or on the news? The media always focus on the single winner but neglect to mention the vast number of losers. Imagine you had a Polaroid of every *loser* in a single lottery drawing.

If we built a box around the Statue of Liberty and wanted to cover the surface of the box with pictures of losers from just one lottery drawing, how much of the box could we cover? (assume a standard Polaroid photo is 3” x 4”). The first step in this exercise would be to calculate the surface area of the box (assume the box has no top or bottom).

Description

Dimensions of the Statue of Liberty:

Statue: height – 151 feet

Base: height – 154 feet
width – 154 feet
depth – 154 feet

The surface area to be covered is = (height of the statue + height of the base) x width of base x 4 sides

(151' + 154') x 154' x 4 = 187,880 sq. ft.

The next step would be to determine how many Polaroid-size photos fit in a square foot:

An Illustration

There are 12 (3”x4”) photos in one square foot. How many square feet of photos do we need to cover the entire scaffolding around the Statue of Liberty? 187,880 square feet.

How many losers is that? 187,880 square feet x 12 losers per square foot = 2,254,560

But don’t we have more losers? 12,271,511 - 2,254,560 = 9,996,991 losers still to paste up on the Statue of Liberty!

If there are 2,254,560 losers on one Statue of Liberty, how many Statues of Liberty could we cover with losers from just one lottery drawing?

$$\frac{\text{12,271,551 losers}}{\text{2,254,560 losers per statue}} = \text{5.44 Statues of Liberty !!!}$$

Can you imagine seeing six Statues of Liberty all in a row?

Can you imagine seeing five of those statues completely covered with pictures of lottery losers from just one drawing — with enough pictures of losers left over to cover the sixth statue up to its waist? Hopefully students will have a new understanding of the vast number of lottery losers for every lottery winner.

Facing the Odds: 

The Mathematics of Gambling and Other Risks

♠ Section 2 Worksheet

1. Based on your own knowledge or opinions, put the following events in order from most likely to least.

____ Being a drowning victim

____ Being killed by lightning

____ Choking to death

____ Winning the lottery (pick 6 out of 48)

____ Being killed in a bicycle accident

____ Dying from a bee sting

____ Being killed by a terrorist in a foreign country

____ Being killed in a car accident

2. Michael plans to use birth dates to select his lottery numbers. Then he remembers that birth dates never go past 31, and the lottery uses numbers 1 through 48. As a result, Michael chooses not to use birth dates because he believes his chances of winning will improve if he selects numbers from the entire range of possibilities. In doing so, is Michael improving his chances of winning? Explain your answer.

3. Erin wants to pick the numbers 1, 2, 3, 4, 5 and 6, but her friend tells her that this is not a good idea because the probability of six consecutive numbers being drawn is so small. Would Erin improve her chance of winning if she chose a more varied six-number combination? Explain your answer.

4. Angela always picks the numbers 4, 8, 15, 17, 23, and 26 for the lottery. While she is away on vacation, her combination is picked. Upon returning, she decides to change her six number combination since it has already been picked. Angela believes that her number combination is less likely to be picked since it was picked only a few days ago. Is she correct? Why or why not?

5. John's sister wins the lottery, and she advises him to stop playing the lottery for a while. She reasons that his chances of winning the lottery have decreased, since the probability of two people in the same family winning the lottery is so small. Is she correct? Why or why not?

6. With any group of 367 people, you are guaranteed that at least two will share a birthday. Why is this true? (Remember that it is possible to be born on February 29 in a leap year.)

7. Indicate which of the following games include dependent events: dice games, coin tossing, card games, the lottery.

Section 2 Worksheet Answers

1. The events, in order from most likely to least likely, are as follows (all of the following probabilities, except for the lottery, are average annual rates based on actual mortality rates; Paulos, 1988, pp. 7, 97):

1. Being killed in a car accident . . . . .one in 5,300
2. Being a drowning victim .one in 20,000
3. Choking to death . . . . .one in 68,000
4. Being killed in a bicycle accident . . . . .one in 75,000
5. Being killed by a terrorist in a foreign country . . . .one in 1.6 million
6. Being killed by lightning .one in 2 million
7. Dying from a bee sting . . . . .one in 6 million
8. Winning the lottery . . . . .one in 12.3 million

None of the other events comes close to the tiny probability of winning a state lottery. Nevertheless, many people probably consider winning the lottery much more likely than, for example, dying in a car crash. And the lottery never publicizes the fact that you are almost eight times more likely to be killed by a terrorist on vacation in Europe than you are to win the lottery.

2. and 3.

Neither player is improving his or her chances of winning. Of the 12.3 million possible 6-number combinations, there are more combinations that have numbers spread out over the entire range than there are combinations that have consecutive numbers or combinations that have numbers all below 32. Thus, the probability that the winning combination will have spread-out numbers is higher than the probability that the winning combination will have all consecutive numbers, or the probability that the winning combination will have numbers all under 32. However, players cannot bet whether the winning number will be "spread out" or "not spread out." They are only allowed to bet that the one 6-number combination they chose will be the one that is randomly selected. Knowing that the winning number is more likely to be "spread out" does not increase their chances of guessing the right number. The combination drawn by the lottery either is their number, or it is not. The probability of any combination being drawn, no matter what it is, is the same as the probability of any other combination being drawn. It does not help to group different combinations by their patterns or characteristics, because in each drawing you are dealing with only one combination.

The following analogy might make this concept more understandable: suppose there were 100 people in your neighborhood, and one person was going to be chosen at random. Your neighborhood is 60% female and 40% male. If we had to bet on what the selected person's gender would be, we would want to bet that the person would be female, since there is a 60% chance that the person will be female and a 40% chance that the person will be male. However, if we were betting that a specific individual would be picked (for example, your mother), that person's gender would have no effect on their chances of being picked. One person is being selected randomly, so each person has an equal chance of being selected. The characteristics of the overall sample are irrelevant if you are dealing with each person as an individual. This example is similar to the lottery in that the category of male/female is similar to the categories of "spread out"/"not spread out" and consecutive/nonconsecutive. You are really dealing with 12.3 million individual outcomes, and the groups have nothing to do with which one is selected. (Items 2 and 3 were taken from Crites, 1994)

The best advice you could give a lottery player would be to stop playing, since the chances of winning are so incredibly small; if someone insists on playing the lottery, one of the few possible intelligent suggestions you could give is that they should pick combinations that they think no one else would pick. Although this technique will not improve their chances of winning, they would be less likely to have to share the jackpot with someone else *if* they won. However, since the vast majority of people will never win, for the vast majority it will be irrelevant whether they follow this advice or not.

4. and 5.

Both Angela and John's aunt are wrong. The outcomes of lottery drawings are not dependent events. Each outcome is random, and previous outcomes do not have any effect on future outcomes. Angela is incorrectly thinking that she is gambling on the outcome of two consecutive events: she is really only betting on the next event. There is a big difference between the following two scenarios:

a. the probability that the next two lottery drawings will both be her number;
b. the probability that the next drawing will be her number, given that she has just won.

Since the random number generator does not remember what combinations it has drawn in the past, previous outcomes do not have any effect on future outcomes. If an event has already happened, you can't use it to calculate the probability of a future independent event. No matter how unlikely it was before it happened, it has a probability of 100% after it has happened: you are guaranteed of the occurrence of that outcome, since it has already happened. Similarly, the random number generator does not know who John is related to, or where he lives, or any other information about him. It is a machine, and it simply draws numbers. John's chances of winning the lottery are the same before or after his sister, or any other person, wins the lottery. (Items 4 and 5 were taken from Crites, 1994.)

6. There are only 366 possible birthdays during the year (365 days plus February 29 on leap year). If you have a group of 367 people, they can't possibly all have different birthdays; if they did, that would make a total of 367 different birthdays, which we know is impossible. Since they can't all have different birthdays, there must be at least two people with the same birthday. To clarify this problem, an analogy can be made with letters and mailboxes: Suppose you have 367 letters and you have to deliver them to 366 mailboxes. After you've put one letter in each mailbox, you still have one letter left over. Since each mailbox already has a letter in it, and you still have to deliver one more letter, one mailbox is going to have two letters in it. Think of the days of the year as 366 mailboxes and the 367 people as the letters: you will see that once every day is occupied by one person, you still have one person remaining, and he or she has to go somewhere. Thus, two people will be in the same "mailbox". Of course, it is possible for several "mailboxes" to have two or more "letters" in them, but we can be *sure* that at least one "mailbox" will have two "letters" in it. (This item was taken from Paulos, 1988.)

7. Only card games involve dependent events: each time a card is removed from the deck, the probability of drawing any particular card or group of cards changes. The rest of the games involve independent events: no outcome or series of outcomes has any effect on future outcomes.