■ PROBABILITY

"Probability theory is the underpinning of the modern world. Current research in both the physical and social sciences cannot be understood without it. Today's politics, tomorrow's weather report and next week's satellites all depend on it."

Darrell Huff

How to Take a Chance, 1959

"It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge … The most important questions of life are, for the most part, really only problems of probability."

Pierre Simon, Marquis de LaPlace

Theorie Analytique des Probabilites, 1812

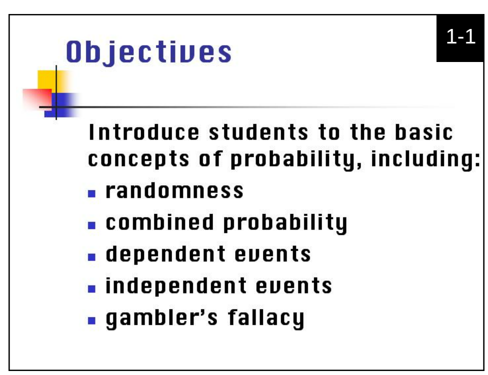
"When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginnings of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science."

William Thompson, Lord Kelvin

♣ ♦ ♥ ♠

Objectives

■ 1-1 The objective for this section is to introduce students to the basic concepts of probability, including randomness, combined probability, dependent and independent events, and the gambler's fallacy.



Concepts of Probability

Introduce probability by defining the concept. Start by asking the students to provide a definition of the word probability. Allow a few answers to get a sense of students' understanding, then provide the definition found in the American Heritage Dictionary of the English Language.

■ 1-2 Probability is:

A. The likelihood that a given event will occur;

B. A number expressing the likelihood that a specific event will occur, expressed as the ratio of the number of actual occurrences to the number of possible occurrences (Soukhanov, 1992).

1-2

Defining Probability

Probability is

A. the likelihood that a given event will occur

B. A number expressing the likelihood that a specific event will occur, expressed as the ratio of the number of actual occurrences to the number of possible occurrences (Soukhanov, 1992)

For example, suppose you were rolling a die and wanted to know how often the number one would come up. The probability is defined in the following equation:

■ 1-3 Probability = (# of favorable results) ÷ (# of possible results) = 1/6. A "favorable result" is the result for which you are determining the probability. In this case, you are determining the probability of a single event (i.e., rolling a 1), so the number of favorable results is one. There are six sides to a die, so there are six possible results. Similarly, the probability that you will roll either a 1 or a 2 on a single roll is 2/6, since there are two favorable results (i.e., a one or a two) and six possible outcomes.

1-3

Probability of rolling a "1"

(# of favorable results) ÷ (# of possible results) = 1/6

A "favorable result" is the result for which you are determining the probability. In this case, you are determining the probability of a single event 1/6 = .17 = 17%

Probability can be expressed as a fraction (1/6 or "one out of six") or a decimal (.17). Fractions are converted into decimals by dividing the numerator by the denominator. In this case, 1÷ 6 = .17. When expressed as a decimal, the probability of an event will always be between 0 and 1, including 0 and 1. An event with a probability of 0 will never happen, and an event with a probability of 1 is sure to happen. For example, the probability that you will roll a 7 with one die is 0; that outcome is impossible. The probability that you will roll a 1, 2, 3, 4, 5, or 6 is 6/6, or 1.0; there are six favorable outcomes and six possible outcomes, so you are guaranteed to get one of the six. For probabilities of 0 and 1, the outcome is always known; for any other probability, you are determining the *likelihood* of an outcome but not what the *specific* outcome on any one trial will be.

Any probability also can be expressed as a percentage. Knowing that the probability of rolling a 1 is 1/6, we can convert this fraction into its decimal form, .17, by dividing 1 by 6. The decimal .17 can also be expressed as 17%. In other words, out of a number of die rolls, you are likely to roll a one 17% of the time. As a percentage, probability ranges from 0% to 100%.

Randomness

Randomness is an important concept to understand when dealing with probability. When we refer to a random process, we mean that each possible outcome in that process has the same chance, or probability, of occurring. In events such as rolling a die and flipping a coin, we make the assumption that all possible outcomes are equally probable.

■ 1-4 For example, when flipping a coin, the probability of the coin landing heads up is the same as the probability of the coin landing tails up (50%). Similarly, when rolling a die, the probability of rolling a one is the same as the probability of rolling a two, which is the same as the probability of rolling any of the possible numbers.

1-4

Randomness

- Each possible outcome in that process has the same chance, or probability, of occurring
- Outcomes are determined by chance

The reason all possible outcomes of events like these are equally likely is that the outcomes are determined by chance; there is no reason to believe that one result should occur more often than another result. Since the factors that determine the outcome on a single roll of a die are unknown and unpredictable, we can say that the outcome on any one roll is random.

Probability in the Real World

For events such as flipping a coin or rolling a die, we can identify all the possible outcomes, and we can assume that (1) the outcome on any one event (e.g., a flip or a roll) is determined by chance and (2) all outcomes are equally probable. For other events in real life, however, one or more of the above conditions probably will not be met.

■ 1-5 For these events that are not so easily analyzed, we can use a different method to determine probability: conduct a series of tests, or "trials," of the event we are interested in and observe the outcomes. Once a sufficiently

1-5

Probability in the Real World

$$\text{Probability} = \frac{\text{\# Favorable Outcomes}}{\text{Total \# Trials}}$$

Probabilities based on trials are an estimate large number of trials have been conducted, we determine the probability of the event in question with the following formula:

Probability = Number of favorable outcomes/Total number of trials

Example

Suppose we want to determine the probability that a basketball player will make a basket from the foul line. Since we cannot assume that all outcomes are equally probable or that any one outcome will be determined by chance, we cannot determine the probability using only a formula: we have to conduct a number of trials.

Suppose we observe the player taking 1,000 consecutive foul shots, and he makes 700 of them. We now have the data necessary to estimate probability:

700 (number of favorable outcomes) ÷ 1,000 (total number of trials) = 70%

From this experiment, we have determined that for each time this particular player goes to the foul line to take a shot, the probability that he will make it is 70%. Unlike probability based on a mathematical formula, probability based on trials should be thought of as an estimate: it is based on previous occurrences, and there is no guarantee that future trials will follow the same pattern. The reason for our uncertainty is that these outcomes are not random: that is, they are not based on chance. If the player was to practice foul shots every day for a month and we were to conduct a number of trials again, the outcome might be different than it was the first time. Similarly, if we observe a player for 30 trials and he makes 30 consecutive foul shots, the probability that he will make the 31st shot is not really 100% — remember that a 100% probability means that the event is guaranteed to happen. In this case, we would probably say that we have not observed a sufficiently large number of trials to calculate an accurate probability.

Combined Probability

So far we have dealt with probability as it applies to the outcome of a single event: for example, what will happen when one player shoots a foul shot. When more than one event is involved, determining probability becomes more complicated.

■ 1-6 For example, instead of flipping a single coin, suppose you were flipping two coins and wanted to determine the probability of *both* coins coming up heads. In other words, you want to know the probability of the first coin coming up heads *and* the second coin coming up heads. The probability that both events will occur is the product of the two separate probabilities:

Probability of 1st *and* 2nd event occurring = (Probability of 1st) x (Probability of 2nd)

Thus, when flipping two coins, the probability of both coming up heads is .5 multiplied by .5, or .25.

What if we wanted to know the probability of one player making two foul shots in a row? Similarly, the probability that the basketball player (referring to example from previous page) will make two foul shots in a row is (.70) (.70) = .49, or, as a round figure, just about 50/50.

Probability of Dependent and Independent Events

When determining the probability of two or more events, it is important to determine whether the outcomes of the events are dependent or independent.

■ 1-7 That is, you must know whether the outcome of any of the events has an effect on the outcome of the subsequent event(s).

1-7

Is an Event Dependent or Independent?

INDEPENDENT
The outcome of the 1st event ***does not*** affect the outcome of the 2nd event

DEPENDENT
The outcome of the 1st event ***does*** affect the outcome of the 2nd event

Example

*If you are flipping a coin twice, the two flips are **independent**: the outcome of the first flip does not affect the outcome of the second in any way.*

Dependent events are events that influence each other's outcomes in some way. With dependent events, the probabilities of the outcomes of some event are affected by the outcomes of previous events.

Example

Here's an example of dependent events: Drawing cards from a deck

Suppose you want to determine the probability of drawing an ace from a well-shuffled deck (that is, the cards are in random order). Since there are 4 aces and 52 cards in all, the probability of drawing an ace on your first try is 4/52 or 1/13 (or approximately 7.7%). If you do not draw an ace on your first try, your chances of drawing an ace on your second try are no longer 4 out of 52: there are still 4 aces in the deck, but you have drawn a card out, so there are only 51 remaining cards to choose from. Thus, as a result of the previous outcome, your chances of drawing an ace have changed to 4/51, or approximately 7.8%. If you had drawn an ace on your first try, your chances of drawing an ace on your second try would be 3/51, or approximately 5.9%. In this card drawing example, each trial removes one of the outcomes from the pool of possible outcomes; once you have drawn a card, that card is no longer in the pool of possible outcomes for future trials. Since each trial affects the potential outcomes of subsequent trials, these events are said to be dependent.

■ 1-8 It is easy to mistake independent events for dependent events. Consider the common misunderstanding known as "the gambler's fallacy."

Example

Gambler's Fallacy

This fallacy reflects the belief that because a coin has come up heads several times in a row, it is more likely to come up tails on the next flip (Paulos, 1988, p.43). In reality, the outcomes are totally independent: the coin does not "remember" what it has done in the past in order to "decide" what it will do next. Or, said differently, an outcome of heads on one trial does not remove heads from the pool of possible outcomes for subsequent trials. No matter what has happened before, the probability of tails for any one coin toss is always 50%.

In dealing with probability, it is very important to know whether you are determining the probability of one event or the probability of a group of events; this is where the inaccuracy of the gambler's fallacy lies.

1-9

Probability of One Event vs. a Group of Events

Assumption: Outcome is Random

Probability of 1st x 2nd x 3rd x 4th =

Combined Probability of a group of events

Example

■ *1-9 Assuming that the chances of being born a boy and the chances of being born a girl are the same (50%) and the outcome is random, the chances of a couple having four girls in a row are 1/2 x 1/2 x 1/2 x 1/2 = 1/16, or about 6%.*

However, if the couple already has three girls, the probability that the next baby will be a girl is 50%. The difference is that in the first example you are dealing with a sequence of four events and in the second you are dealing with a single event.

If you are dealing with a trial consisting of a single event, you cannot look at random events that have occurred previously and tie them in with the probability of the event in question. If you are dealing with a trial consisting of four events (four coin tosses, four consecutive children), the probability in question is how many times the favorable sequence of four events will come up in a number of four-event trials. For example, out of 100 couples that have four children, how many will have four girls? About 6 couples (6% for group of events). Another way to look at this situation is to evaluate it at the two different points of time in question: if a couple plans to have four children, what are the chances that all will be girls? One in sixteen (1/16 for group of events). If another couple already has three girls, what are the chances that they will end up with four girls? One in two (1/2 for single event). Clearly, it is very important to frame the probability question correctly: there is a big difference between determining the probability of one event and determining the probability of four events.

If we return to the example of the gambler's fallacy, we can see how these concepts apply. We know that the probability of getting four heads in a row is 1/2 x 1/2 x 1/2 x 1/2 = 1/16. However, if we have just flipped three heads in a row, and we want to determine the probability of heads on the next flip, it doesn't matter what the probability of getting four heads in a row is: at this point, we are only dealing with the *next* flip. In other words, we are really dealing with only one flip, not four. Since these events are independent, it doesn't matter what has happened in the past. Thus, although determining the probability of a certain outcome for a *group* of coin flips is different from determining the probability of a certain outcome with *one* coin flip, each individual outcome is independent.

EXERCISES

Legend: For Use in Section Exercises

Directions or Descriptions

Double line boxes like this are for directions or descriptions; the typeface is italicized.

Example

Boxes like this are for examples to be used for reinforcement of the concept presented.

Classroom Discussion

Boxes like this contain information for classroom discussion.

An Illustration

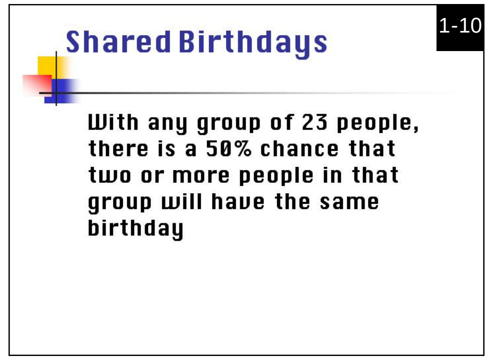
Boxes like this contain information that can be used to illustrate a point.

Derivation or Calculation

Boxes like this are used when information is available to show the derivation of a formula or to demonstrate the calculations made in pursuit of a solution.

♣ Shared Birthdays

■ 1-10 With any group of 23 people, there is a 50% chance that two or more people in that group will have the same birthday (Paulos, 1988, p. 27; McGervey, 1986, p. 9).



Directions

This exercise incorporates this phenomenon into a classroom exercise.

1. *Assign one student from each homeroom in your grade to survey the students in his or her homeroom class to determine the birthdays of each class member.*
2. *Have each homeroom representative report back to you the birthdays of each student in his or her class.*
3. *As a class exercise, identify the classes in which two or more students share a birthday.*
4. *Then determine what percentage of the classes had a shared birthday.*

Example

If there were 6 homerooms in your grade, you would assign one person from each homeroom to survey their homeroom class members. Each of these 6 students would then hand in a report listing each student's birthday. You might want to break the class into six groups, give each group a list of the birthdays of a homeroom's members, and have the group determine whether two or more students share a birthday. Then have each group report to the class whether there was one or more shared birthday on their list. To determine the percentage of homerooms that had a shared birthday, divide the number of homerooms with a shared birthday by the total number of homerooms.

$$\frac{\text{\# homerooms with shared birthday}}{\text{total \# homerooms}} = \text{\% homerooms with shared birthday}$$

Classroom Discussion

Discuss the outcome with your class.

- Was the result close to 50%?
- If not, ask the students why they think it was not.

Here are a couple of reasons why your outcome might be different from 50%:

1. The homerooms had more or fewer than 23 people. If there were fewer than 23 in a homeroom, the chances of having a shared birthday would be less than 50%. If there were more than 23, the chances would be greater than 50%.

2. This exercise is similar to tossing a coin. Just as there is a 50% chance of getting heads and a 50% chance of getting tails on any one flip, there is a 50% chance of having a shared birthday and a 50% chance of having no shared birthdays in any one group of 23 people. With a large number of trials, the outcomes will begin to approximate 50/50, but with a small number of trials the outcomes are less likely to approximate the true probability.

An Illustration

To illustrate this point, you could have the students do one coin flip for each homeroom- if there are six homerooms, they would flip a coin six times. The 50/50 probability of each coin flip would represent the 50/50 probability of having a shared birthday in each of the homerooms. Next, calculate the percentage of the flips that came out heads. Is it close to 50%? The point of this comparison between the homerooms and coins is that although the probability dictates that heads will come up 50% of the time, this is not always what happens with a small number of this.

Derivation

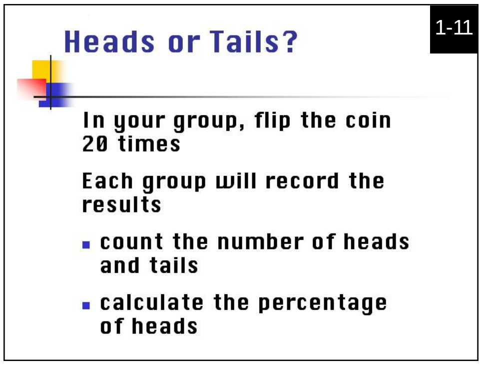
Suppose you had 23 people lined up in a row. First you identify the first person's birthday. The probability of the second person's birthday being different from the first would be 364/365 (exclusive of the leap year). The probability that the third person would have a birthday different from the first and second is 363/365. The probability that the fourth person would have a birthday different from the other three is 362/365. This pattern continues until you reach the 23rd person.

Using the following equation: 364/365 x 363/365 x 362/365 x … x 343/365 = .4927.

The result 49.27% represents the probability that all 23 people will have a different birthday. The alternate possibility, that at least two of the people will have the same birthday, has a probability of 50.73%.

♣ Heads or Tails?

■ 1-11



Directions

1. *Divide the students into groups of two or three.*
2. *Give each group a coin.*
3. *Have each group flip its coin 20 times and have them record the results.*
4. *When they are done, have each group count the number of heads and tails.*
5. *Record the results for each group on the blackboard.*
6. *Calculate the percentage of heads for each group (# of heads/20).*
7. *Make note of how close each group is to 50%.*
8. *Next, calculate the total number of heads flipped and the total number of tails.*

Classroom Discussion

Lead the class in a discussion of the outcomes using the following questions:

- What percentage of the class total was heads?
- How does the percentage of heads in the class total compare with the percentages of heads in the various groups? (Is it generally closer to or farther away from 50%?)
- Why does this pattern occur?

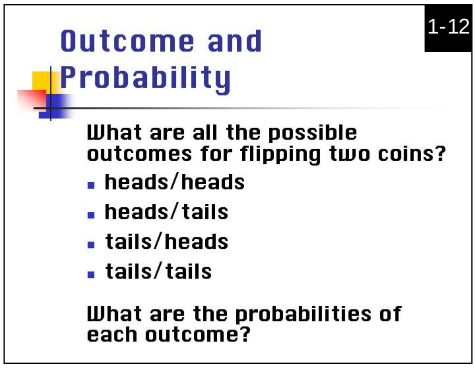
If the outcome of each coin toss is random and the probability is exactly 50% for heads and tails, why didn't the class come up with exactly the same percentage of heads and tails?

Directions

Generally, the smaller groups will vary more widely than the overall group. That is, the small groups will be farther away from 50% than the overall total will be. With any event with a random outcome, the true outcome will get closer to the expected probability (in this case, 50%) as the number of trials increases. With any small number of trials (in this case, 20), you could get a greater number of heads or tails by chance, even though the probability of each is 50%. With a large number of trials, however, the groups of 20 that had a larger number of heads will approximately balance out the groups that had a larger number of tails. With a random event, you never know what the outcome of any one trial (or a small group of trials) will be, but you can be fairly confident that the real outcome will approximate the expected probability in the long run.

♣ Outcome and Probability

■ 1-12



Directions

Without giving the students coins, have them determine all of the possible outcomes for flipping two coins.

- *Allow a couple of minutes for students to write their answers, then proceed.*

Classroom Discussion

Ask students to determine the probability of each outcome. The four possible outcomes of flipping two coins are:

- heads/heads
- heads/tails
- tails/tails
- tails/heads

For each of the four possible outcomes, the probability is .5 X .5 = .25.

The students may not initially understand that heads/tails is a different outcome than tails/heads. They should think of the coins as two separate coins (first coin/second coin, penny/nickel, or their coin/a friend's coin). To reinforce this point, have them calculate the probability that the two coins will have the same side showing and the probability that the two coins will have different sides showing. In each case, the probability is 50%. You can calculate this with the following formula:

- (# of favorable outcomes) ÷ (# of possible outcomes).

In two of the four possible outcomes, the same side is showing, and in the other two outcomes, different sides are showing.

♣ Probability in the Real World: Becoming a Legend

■ 1-13

Directions

Present the following story to the class:

In the history of baseball, a season batting average of .400 has been one of the major achievements for a hitter to attain. Batting average for a season is calculated by dividing the player's number of hits during the season by the number of times that the player has been at bat.

A .400 batting average means that the player has gotten a hit 4 times out of every 10 times at bat. In 1941, Ted Williams, the legendary Red Sox slugger, was approaching the end of the season with nearly a .400 batting average. On the very last day of the season, the Red Sox were playing a doubleheader (two games in one day) against the Philadelphia Athletics.

During the entire season, Ted Williams had gotten 179 hits out of 448 times at bat, for a batting average of .39955 (Linn, 1993). Since batting averages are only carried to three places, this batting average would be rounded off to .400. Since Williams risked falling below .400 if he played in the last two games, the Red Sox coach offered to let Williams sit the games out. "No," Williams said, "I'm going to play. If I'm going to be a .400 hitter, I'm not going to slip through the back door, and I'm not going to do it sitting on a bench" (Linn, 1993, p. 160).

■ 1-14

1-14

Becoming a Legend: Part II

- Given Ted Williams batting average before the last day (.39955), how many hits would you expect him to get out of 8 times at bat?
- If Williams went to bat 8 times on his final day and got the number of hits calculated above, what would his season average have been?
- Would you have played the last games or sat them out?

Classroom Discussion

1. Given Ted Williams' batting average before the last day (.39955), how many hits would you expect him to get out of 8 times at bat?
2. If Williams went to bat 8 times in his final day and got the number of hits calculated in #1 above, what would his season average have been?
3. Would you have played the games or sat them out?

Description

Assuming Williams played in the last two games as he had in all the previous games that season, he would have gotten 3 hits out of 8 times at bat (.39955 ÷ 8 = 3 hits).

If he had gotten 3 hits out of 8 times at bat on the last day of the season, he would have had a season total of 182 hits out of 456 times at bat, or a .399 average. To maintain his .400 average, he would have needed 4 hits out of 8 times at bat, or an average of .500 for the day. However, in those two games, Williams got 6 hits out of 8 times at bat, and finished the season with a .406 batting average. Only a handful of players have equaled this achievement in the history of baseball. Williams remains the last player ever to have hit .400 (background from Linn, 1993).

Having heard this story and calculated these figures, students should appreciate the risk Williams was taking on the last day of the 1941 season. Discuss with the students whether they think it was wise for Williams to play, and what they would do if they were in a similar position. Was Williams gambling when he played the last two games? Discuss the risk he took, focusing on the distinction between risks based on skill (e.g., baseball batting average) and risks based on chance (i.e., gambling that involves no skill, such as a coin toss).

♣ You Bet Your Life

This exercise is based on a section with the same title in McGervey (1986), Probabilities in Everyday Life.

Description

The primary objective of this exercise is to teach probability through another real-life example. A secondary objective is to remind students of the risks involved in smoking or chewing tobacco. This exercise is a symbolic one; the probabilities of the outcomes presented in this exercise are not exact. This exercise will help students understand the process of making real-life choices on the basis of probability.

Directions

Tobacco-related diseases caused a quarter of all deaths in the U.S. in 1990. Each year more than 400,000 people die as a result of smoking. The average age that habitual smoking begins is currently 14.5 years; approximately 90% of regular smokers start before the age of 21 (Bartecchi et al, 1995). "Cigarette smoking has been identified as the most important source of preventable morbidity and premature mortality in the United States" (Bartecchi et al, 1994, p. 907).

Ask students if they can name any diseases related to smoking. Make a list on the board. Tobacco-related diseases include numerous cancers, cardiovascular diseases, and pulmonary illnesses (e.g., emphysema, pneumonia). Listed in the table on the following page are many of the conditions known to be associated with tobacco (Bartecchi, MacKenzie, & Schrier, 1995).

Examples

Cancer:
- Lung Cancer
- Cancer of pharynx
- Cancer of larynx
- Cancer of esophagus
- Cancer of stomach
- Cancer of pancreas
- Cancer of uterine cervix
- Cancer of kidney
- Cancer of ureter
- Cancer of bladder
- Cancer of colon
- Leukemia

Cardiovascular Disease:
- Stroke
- Sudden death
- Heart attack
- Peripheral vascular disease
- Aortic aneurysm
- Atherosclerosis

Pulmonary Illness:
- Pneumonia
- Emphysema
- Bronchitis
- Influenza
- Chronic airway obstruction

Reproductive Risks:
- Reduction of fertility
- Spontaneous abortion & stillbirth
- Lower birth weight in infants
- Cervical cancer

Other:
- Cataracts
- Delayed healing of broken bones
- Periodontal maladies
- Ulcers
- Hypertension
- Brain hemorrhage
- Wrinkles

Classroom Discussion

Some smokers will get one of these diseases, some will get more than one of these diseases, and some smokers will get none. Some of the students will have heard about the 104-year-old woman who has smoked, drank, and "had a good time" all her life and is still healthy and happy. She is an example of an individual who engaged in behaviors that have a high probability of leading to disease, but who "beat the odds." In addition, it is rare to hear of an adolescent with lung cancer. The result is that most adolescents think they, too, will beat the odds. "Adolescents have a sense of invulnerability — an attitude of 'it won't happen to me'" (Dusenbury, Khuri, & Millman, 1992, p. 833). An adolescent who is currently healthy tends to think long-term health consequences are too far off in the future to be real or threatening. As Paulos (1995) puts it, "suffering ordained for twenty years from now is, like a million-dollar debt due in twenty years, considerably easier to bear than is suffering scheduled for tomorrow."

Group Activity

- *Divide the class into small groups (4 or 5 students each).*
- *Give each group a pair of dice.*
- *Have each group designate one student to be the record-keeper. Give a copy of the Hands-on Activity sheet for "You Bet Your Life" to each record-keeper.*
- *Tell the class that each member of each group will take a turn rolling the dice. Each student in the group will roll the dice one time while the record-keeper enters outcomes and their corresponding disease in each row in the table. As the record-keeper records the outcome, he or she should inform the dice-roller whether he or she will "contract a disease" and, if so, which one by referring to the table on page 46.*
- *When all the groups have finished, ask them to tally their results in the box. Students should tally how many times each disease (and no disease) was contracted.*
- *The teacher will debrief. An example follows on the next page.*

Directions

The teacher should display the data in a summary format on the board. The following chart is one possible summary method, filled with example data for 5 groups of 4 students each.

Sample Summary Chart

Disease	Totals from each group 1 2 3 4 5	Total # with Disease/ Total in Class	Percentage "With Disease"
No disease – healthy	1 + 0 + 1 + 0 + 0	2/20	10%
Cancer of the Lips	1 + 0 + 0 + 1 + 1	3/20	15%
Cancer of the Mouth	0 + 0 + 1 + 0 + 0	1/20	5%
Stroke	1 + 0 + 0 + 0 + 1	2/20	10%
Emphysema	0 + 1 + 0 + 1 + 0	2/20	10%
Lung Cancer	1 + 1 + 2 + 0 + 1	5/20	25%
Bronchitis	0 + 1 + 0 + 1 + 0	2/20	10%
Heart Disease	0 + 0 + 0 + 1 + 0	1/20	5%
Ulcers	0 + 1 + 0 + 0 + 0	1/20	5%
Wrinkles	0 + 0 + 0 + 0 + 1	1/20	5%
Cancer of the Tongue	0 + 1 + 0 + 0 + 0	1/20	5%

This model assumes that each student will get, at most, one disease, which is a simplified and conservative estimate of risk for smokers. However, the objective of this exercise is for students to understand that there are varying probabilities for contracting different diseases. Why did lung cancer have the greatest percentage among the diseases those students "contracted?" If the summary table fails to match the theoretical distribution, you can increase the number of throws. The following table demonstrates the probabilities of rolling the eleven possible totals of 2 dice. Because there are more possible ways to roll a 7, for example, the outcome probability is highest for a roll of 7. There is only 1 way to roll a 2 or an 12, so the probability for both of these outcomes is 1/36, or .027.

Derivation of Probability Outcomes

1st die	2nd die	Total of 2 Dice	Outcome Prob.	Disease
1	1	2	1/36	NO DISEASE
1	2	3	2/36	Cancer of the Lips
2	1			
1	3	4	3/36	Cancer of the Mouth
2	2			
3	1			
1	4	5	4/36	Stroke
2	3			
3	2			
4	1			
1	5	6	5/36	Emphysema
2	4			
3	3			
4	2			
5	1			
1	6	7	6/36	Lung Cancer
2	5			
3	4			
4	3			
5	2			
6	1			
2	6	8	5/36	Bronchitis
4	4			
5	3			
6	2			
3	5			
3	6	9	4/36	Heart Disease
4	5			
5	4			
6	3			
4	6	10	3/36	Ulcers
5	5			
6	4			
5	6	11	2/36	Wrinkles
6	5			
6	6	12	1/36	Cancer of the Tongue

Example

Student	Total Both Dice	Tobacco-Related Disease
#1		
#2		
#3		
#4		
#5		
#6		
#7		

Example

2 = Healthy: No disease

3 = CL: Cancer of the lips

4 = CM: Cancer of the mouth

5 = ST: Stroke

6 = EM: Emphysema

7 = LC: Lung cancer

8 = BR: Bronchitis

9 = HD: Heart disease

10 = UL: Ulcers

11 = WR: Wrinkles

12 = CT: Cancer of the tongue

Example

Results

Disease	Total
Healthy	
CL	
CM	
ST	
EM	
LC	
BR	
HD	
UL	
WR	
CT	

Facing the Odds:

The Mathematics of Gambling and Other Risks

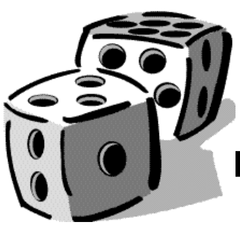
♠ Section 1 Worksheet

The examples used in this worksheet are not intended to be morbid or frightening. The use of low base rate occurrences is necessary to illustrate the lessons contained in this module. Infrequent, relatively random events, by their nature, are usually associated with tragedy.

1. If you are rolling a die, what is the probability that you will roll a six? Write your answer in fraction, decimal, and percent form.

2. What is the probability that you will roll either a four or a five?

3. **a.** What is the probability of rolling an even number?

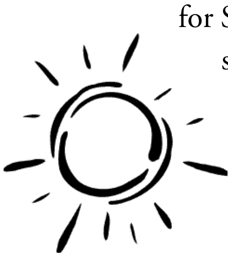


b. What is the probability of rolling an odd number?

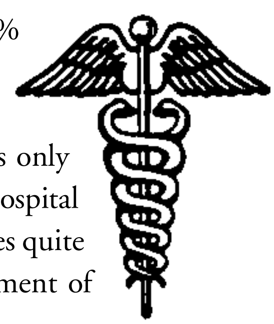
4. If you are rolling two dice, what is the probability of rolling a two on both?

5. Your favorite basketball team is down by one point. Time has run out in the fourth quarter, and your team has a player at the foul line for two shots. You know that the player has made 80% of the foul shots he has taken this season. What is the probability that your team will win the game?

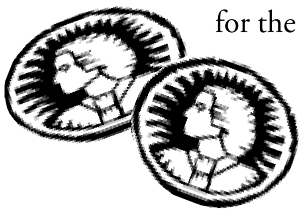
6. On Friday night, the weather forecaster says that there is a 99% chance of rain for Saturday. Saturday turns out to be a beautiful sunny day. Can you say that the weather forecaster's prediction was wrong? Why or why not?



7. Recently you have been diagnosed with a minor physical ailment. You are considering having your doctor treat your condition with a medical procedure, but you are not sure how safe this procedure is. To determine whether the chance of treatment is worth the risk of complications, you make an appointment to speak with your doctor about it. " Don't worry," your doctor tells you, "this procedure is 99% safe." You mention that you had heard that there can be complications with this procedure, and you express your reservations. "Complications are a possibility," your doctor replies, "but the risk is only one-in-a-million." You ask your doctor how often patients in the hospital where he works have problems with the procedure. "Oh, it usually goes quite well," he answers. Can you find a problem with your doctor's assessment of this procedure's risk?



8. Two friends of yours are betting on the outcome of coin tosses. One of them always likes to bet on tails, and tails has come up three times in a row. She decides to change her bet for the next toss. She reasons that after three tails, a heads is "due." What is your advice?



a. What are the chances of drawing a queen from a complete deck of shuffled cards?

b. What are the chances of drawing the queen of hearts?

c. If you have drawn two cards and neither one is a five, what are the chances that you will draw a five the next time?

d. If you have drawn a four and a five, what are the chances of drawing a five the next time?

9. A friend of yours has a cousin who was in a minor plane crash and survived. Whenever your friend has to fly in an airplane, she insists on being accompanied by her cousin, figuring that the probability of her cousin being in two plane crashes is very small. Is your friend's reasoning correct?

10. An acquaintance of yours is concerned about a recent terrorist bombing of an airline flight that he read about in the paper. He decides that on all future flights he takes, he will bring a bomb in his suitcase, reasoning that the probability of two bombs being on a plane is very, very low. Will his plan decrease his chances of being killed by a terrorist?

Section 1 Worksheet Answers

1. The probability that you will roll a six is 1/6, .1667, or 16.67%

2. The probability that you will roll either a four or a five is 2/6, or 1/3: there are two favorable outcomes and six possible outcomes. This probability can also be expressed as .3333 or 33.33%.

3. Since there are three even numbers on a die 2, 4, and 6, the probability is 3/6, or 1/2 .50 or 50%.

4. The probability of rolling a two on the first die is 1/6. The probability of rolling a two on the second die is 1/6. So the probability of both happening is 1/6 x 1/6, or 1/36 .0278 or 2.78%.

5. The probability that he will make the first shot is 80%, and the probability that he will make the second shot is 80%, so the probability that he will make both is .80 x .80 = .64. Your team has a 64% chance of winning.

6. You can't necessarily say that the weather forecaster is wrong. With a 99% chance of rain, there is still a 1% chance of sun. No matter how probable rain is, it is still possible to get sun. In other words, a probability only tells you the likelihood of an outcome, not exactly what the outcome is going to be.

7. Your doctor has reported three significantly different levels of risk. The first time, he said that the procedure is 99% safe, meaning 99 times out of 100. In other words, 1 person out of 100 will have complications, or 10,000 out of a million. The second time, he said that the procedure has a one-in-a-million risk, which means that one person out of a million will have complications, which is a lot better than the ten thousand out of a million risk he reported the first time. The third time, he said that the procedure usually goes well, which doesn't give you any specific data on which to base your decision. At the least, you can assume "usually" means that the procedure goes well more often than it doesn't go well — in other words, that it goes well at least 51% of the time and goes badly at most 49% of the time. At this level of risk, as many as 490,000 people out of a million will have complications. Clearly, this doctor does not understand probability — he does not know whether 1, 10,000 or 490,000 people out of a million will have problems with this procedure. This example was adapted from Paulos (1988).

8. You should tell your friend that it doesn't matter whether she changes her bet or not — either way, she can't improve her chances of winning. No matter what has happened before, the outcome of coin tosses is always random and the probability for heads is the same as the probability for tails. Coin flips, like dice rolls and lottery drawings, are not dependent on previous outcomes. That is, previous outcomes do not affect current or future outcomes in any way.

a) 4/52 = 1/13 = 7.69%; b) 1/52 = 1.92%; c) 4/50 = 8%; d) 3/50 = 6%.

9. Your friend's reasoning is not correct. Plane flights are not dependent events: in other words, the outcome of one flight does not have an effect on the outcome of subsequent flights. The airplanes used in commercial flights do not "remember" that someone was in a crash and thus exempt that person from further crashes. Assuming that the probability of a commercial flight crashing is one in a million McGervey (1986) a person will be subjected to that one-in-a-million risk each time he or she takes a commercial flight, whether he or she has been in a previous crash or not. In reality, there are specific causes of plane crashes: faulty or damaged airplane parts, engine failures, weather conditions, pilot incompetence, etc. However, since you cannot evaluate all of the pertinent variables each time you want to take a flight, you have to assume that plane crashes are more or less random occurrences. That is, since the factors leading to a crash are unknown and unpredictable before the crash, anyway, you have to approach crashes as rare chance outcomes. Clearly, the presence or absence of someone who has been in a previous crash does not influence the outcome of the flight in any way.

10. Your friend's plan will not change his chances of being blown up by a terrorist. Let's say, hypothetically, that the chance of a terrorist planting a bomb on any particular plane is one in 5 million. The infinitesimal probability your friend is thinking of is the probability that two different terrorists will each plant a bomb on his plane independently, resulting in two bombs on his plane. Indeed, that probability is infinitesimal. However, his plan does not change the probability of a terrorist planting a bomb on his plane. If a terrorist choosing to blow up the plane that he happens to be on is a chance occurrence, then what he is carrying in his suitcase does not affect whether it will happen or not. Mathematically, we know that the probability of two specific outcomes occurring is probability of outcome 1 x the probability of outcome 2. If your friend knows that he will be carrying a bomb on a particular flight, then he knows that the probability of his having a bomb is 1: he is 100% confident of that outcome occurring. To determine the probability of his having a bomb *and* a terrorist having a bomb on his plane, he must multiply the two probabilities: 1 x 1/5,000,000, which equals 1/5,000,000. In other words, his plan has no effect. This example was adapted from Paulos (1988).